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R.U. 71
Mith. U
8th

Q.No → Test the Convergent of series $\sum \{\sqrt{n^4+1} - n^2\}$.

Ans → Let the nth term of the given series be denoted by u_n

$$u_n = \sqrt{n^4+1} - n^2$$

$$= \frac{(\sqrt{n^4+1} - n^2)(\sqrt{n^4+1} + n^2)}{\sqrt{n^4+1} + n^2}$$

$$= \frac{n^4 + 1 - n^4}{\sqrt{n^4+1} + n^2} = \frac{1}{\sqrt{n^4+1} + n^2}$$

$$= \frac{1}{n^2\left(\sqrt{1+\frac{1}{n^4}} + 1\right)}$$

Let us Consider an Auxiliary series whose nth term $v_n = \frac{1}{n^2}$

$$\frac{u_n}{v_n} = \frac{1}{\sqrt{1+\frac{1}{n^4}} + 1}$$

$$\therefore \lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{1}{n^4}} + 1} = \frac{1}{\sqrt{1+\frac{1}{\infty^4}} + 1} = \frac{1}{\sqrt{1+0}+1} = \frac{1}{2}$$

which is finite and non zero

Hence, from Comparision test $\sum u_n$ and $\sum v_n$ will be Convergent and divergent simultaneously

But $v_n = \frac{1}{n^2}$ which is Convergent

Hence, by Comparision test

$\sum u_n$ is also Convergent.

(26) Q.N. → Test the convergency of the series whose general term is

M.U. 75, 83

$$\sqrt{n^4+1}-\sqrt{n^4-1}.$$

Ans. → Let the given nth term of the series be denoted by u_n

$$u_n = \sqrt{n^4+1} - \sqrt{n^4-1}$$

$$= \frac{(\sqrt{n^4+1} - \sqrt{n^4-1})(\sqrt{n^4+1} + \sqrt{n^4-1})}{(\sqrt{n^4+1} + \sqrt{n^4-1})}$$

$$= \frac{\cancel{n^4}+1-\cancel{n^4}+1}{\sqrt{n^4+1}+\sqrt{n^4-1}} = \frac{2}{\sqrt{n^4+1}+\sqrt{n^4-1}}$$

$$= \frac{2}{n^2\left(\sqrt{1+\frac{1}{n^4}}+\sqrt{1-\frac{1}{n^4}}\right)}$$

Let us consider an Auxiliary series whose nth term

$$v_n = \frac{1}{n^2}$$

$$\frac{u_n}{v_n} = \frac{\frac{2}{n^2\left(\sqrt{1+\frac{1}{n^4}}+\sqrt{1-\frac{1}{n^4}}\right)}}{\frac{1}{n^2}} = \frac{2}{\cancel{n^2}\left(\sqrt{1+\frac{1}{n^4}}+\sqrt{1-\frac{1}{n^4}}\right)} \times \frac{\cancel{n^2}}{1}$$

$$\therefore \lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty} \frac{2}{\left(\sqrt{1+\frac{1}{n^4}}+\sqrt{1-\frac{1}{n^4}}\right)} = \frac{2}{\left(\sqrt{1+\frac{1}{\infty^4}}+\sqrt{1-\frac{1}{\infty^4}}\right)}$$

$$= \frac{2}{\sqrt{1+0}+\sqrt{1-0}} = \frac{2}{2} = 1,$$ which is finite and non zero

Hence, from comparision test Σu_n and Σv_n will be convergent and divergent simultaneously.

But $v_n = \frac{1}{n^2}$ which is convergent

Hence, by comparision test

Σu_n is also convergent.

27 P.U. 65 | Q.No → Test for convergence the series, whose nth term is

$$\sqrt{n^5+1}-\sqrt{n^5}.$$

Ans. → Let the given nth term of the given series be denoted by u_n

$$u_n = \sqrt{n^5+1}-\sqrt{n^5}$$

$$= \frac{(\sqrt{n^5+1}-\sqrt{n^5})\cdot(\sqrt{n^5+1}+\sqrt{n^5})}{\sqrt{n^5+1}+\sqrt{n^5}}$$

$$= \frac{\cancel{n^5}+1-\cancel{n^5}}{\sqrt{n^5+1}+\sqrt{n^5}} = \frac{1}{\sqrt{n^5+1}+\sqrt{n^5}}$$

$1+\frac{1}{2} = \frac{2+1}{2}$

$$= \frac{1}{\sqrt{n^5}\sqrt{1+\frac{1}{n^5}}+1} = \frac{1}{n^{1/2 \cdot 5}\sqrt{1+\frac{1}{n^5}}+1}$$

$$= \frac{1}{n^{5/2}\sqrt{1+\frac{1}{n^5}}+1}$$

Let us consider an Auxiliary series whose nth term:

$$v_n = \frac{1}{n^{5/2}}$$

$$\frac{u_n}{v_n} = \frac{\dfrac{1}{n^{5/2}\sqrt{1+\frac{1}{n^5}}+1}}{\dfrac{1}{n^{5/2}}} = \frac{1}{\cancel{n^{5/2}}\sqrt{1+\frac{1}{n^5}}+1}\times\frac{\cancel{n^{5/2}}}{1}$$

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty}\frac{1}{\sqrt{1+\frac{1}{n^5}}+1} = \frac{1}{\sqrt{1+\frac{1}{\infty^5}}+1} = \frac{1}{\sqrt{1+0}+1} = \frac{1}{2}$$

which is finite and non zero

∴ from comparision test Σu_n and Σv_n will be convergent and divergent simultaneously

But $v_n = \frac{1}{n^{5/2}}$ which is convergent

Hence, by comparision test

Σu_n is also convergent.

28\. Q.No → Test for convergence the series, whose nth term is

$$\frac{\sqrt{n+1}-\sqrt{n}}{n^p}:$$

Ans. → Let the nth term of the given series be denoted by u_n.

$$u_n = \frac{\sqrt{n+1}-\sqrt{n}}{n^p}$$

$$= \frac{\sqrt{n+1}-\sqrt{n}}{n^p} \cdot \frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}}$$

$n\sqrt{n}$
$n^{p+1/2}$

$$= \frac{\cancel{n}+1-\cancel{n}}{n^p(\sqrt{n+1}+\sqrt{n})} = \frac{1}{n^p(\sqrt{n+1}+\sqrt{n})}$$

$$= \frac{1}{n^{p+1/2}\left(\sqrt{1+\frac{1}{n}}+1\right)}$$

Let us consider an Auxiliary series whose nth term

$$v_n = \frac{1}{n^{p+1/2}}$$

$$\therefore \frac{u_n}{v_n} = \frac{\dfrac{1}{n^{p+1/2}\left(\sqrt{1+\frac{1}{n}}+1\right)}}{\dfrac{1}{n^{p+1/2}}} = \frac{1}{\cancel{n^{p+1/2}}\left(\sqrt{1+\frac{1}{n}}+1\right)} \times \frac{\cancel{n^{p+1/2}}}{1}$$

$$\lim_{n\to\infty}\frac{u_n}{v_n} = \lim_{n\to\infty}\frac{1}{\left(\sqrt{1+\frac{1}{n}}+1\right)} = \frac{1}{\sqrt{1+\frac{1}{\infty}}+1} = \frac{1}{1+0+1} = \frac{1}{2}$$

which is finite and non zero

$\therefore$ from Comparision test $\sum u_n$ and $\sum v_n$ will be convergent and divergent simultaneously

But $v_n = \dfrac{1}{n^{p+1/2}}$

$\because$ v_n is convergent, when, $p+\frac{1}{2}>1$, i.e. $p>\frac{1}{2}$

and v_n is divergent when $p \leq \frac{1}{2}$

Hence, the given series is convergent when $p>\frac{1}{2}$ and divergent, when $p\leq \frac{1}{2}$